Mathina

AN INTERACTIVE STORYBOOK BETWEEN MATHEMATICS AND FANTASY

MATHINA'S WORLD

https://imaginary.org/project/mathina

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1 INTRODUCTION

WELCOME TO MATHINA'S WORLD: the world of math-inspired stories and games!

Mathina is an Erasmus+ project, launched by German, Finnish, Italian, Portuguese, and Slovenian organizations in 2018 for the playful and creative development of mathematical skills and competences. Each story in Mathina's world includes puzzles, which develop problem-solving strategies in various mathematical topics.

Stories and puzzles in Mathina's world focus on four thematic areas:

- Logical thinking
- Spatial visualization
- Cryptography
- Symmetry

Each topic offers exciting math&fun activities for different age groups:

- 4-6
- 7-10
- 11-14
- 15-19+

1.1 Learning Mathematics through Stories with Mathina

One of the main purposes of mathematics learning is to let everyone experience the joy of thinking. Children's joy grows with each discovery, and mathematics indeed consists of interesting problems to think about. "It's an enormous experience to understand how the world works," says Kyoto-prize awarded mathematician, László Lovász. Solving a difficult problem might require thinking and testing various solutions for hours, days, weeks, months, years, and even a whole lifespan. It requires persistence and willpower. It requires also leaving the comfort zone, turning into possible what seemed impossible, thinking outside of the box. Therefore, mathematical problem solving is not simply a pragmatic process, but it expands the imagination, develops creativity, and can also improve your character. One can practice how to face the unexpected and not give up even in seemingly hopeless situations.

When we encounter a problem or an idea, we think. It would be useful to better understand the nature of thinking, including our own thinking. George Pólya, who was a founder of the modern methodology of mathematical problem-solving, in his famous work, How to Solve It, argues that learning to think can be efficiently practiced and developed through mathematical models.

Raising attention to mathematical models requires interesting problems. Mathematical thinking always has a purpose. Curiosity can be awakened by full emotional involvement in the thinking and learning process.

Problem-solving can be engaging, motivating and a lot of fun when it is part of an exciting story or an amusing game. The MATHINA project offers a combination of all of these.

Introducing LOGICAL THINKING, SPATIAL VISUALIZATION, CRYPTOGRAPHY, and the overarching topic of SYMMETRY in the form of exciting stories and fun games, takes us beyond the walls of a traditional classroom. Let's leave the limits of subject-based mathematics learning behind!

We hope that the journey into Mathina's story-world will boost creativity and innovative thinking, develop curiosity, imagination, problem-solving, critical perspectives and positive risk-taking.

1.2 Mathina in a transforming education: skills and competences

Education today is in transformation everywhere around the world. Traditional ways of teaching and learning are contested by environmental challenges, social changes, massive information and communication networks, and highly interconnected technologies. Accelerating innovation and the growing needs of the global economy reshape our learning environments from one day to the next.

1 Interview with László Lovász. Index 2017.12.05, (in Hungarian). Accessed online: [03.01.2020] https://index.hu/tudomany/2017/12/05/amit_az_ember_nem_ert_attol_fel/

2 COUNCIL RECOMMENDATION of 22 May 2018 on key competences for lifelong learning. Official Journal of the European Union. 2018/C 189/01. p. 7-8. Accessed online: [03.01.2020] https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32018H0604(01)&from=EN

To keep pace with the change, the European Union's policies in education put emphasis on the life-long development of skills and competences. Mainly those, which support everyone in finding fulfilling jobs and becoming independent, active citizens.

The European Council's Recommendation defines eight key competences for lifelong learning:

- Literacy competence
- Multilingual competence
- Mathematical competence and competence in science, technology and engineering
- Digital competence
- Personal, social and learning to learn competence
- Citizenship competence
- Entrepreneurship competence
- Cultural awareness and expression competence

When mathematics education is focusing on learning to think through problem-solving, mathematical skills and competencies have great potentials to be developed in close interaction with every other competence field.

The Mathina project

- approaches mathematical skills and competences as gateways to other competence fields and vice versa
- supports both individual and collaborative learning
- supports the diversity of learning styles
- welcomes new ideas
- offers multiple stories for multiple perspectives
- recognizes that attitudes towards mathematics are socially constructed and developed often at the same time with mathematical knowledge and skills

In Mathina's world all learners are adventurers, who can actively participate in the discoveries and autonomously reflect on their own learning journey.

1.3 Combining Learning and Entertainment in Mathina's World

Education and entertainment seem to belong to different worlds when we look at traditional learning methods. However, today's students usually find learning in traditional settings unappealing, and require something more in exchange for their attention. In Mathina we would like to make young people enjoy the process of learning. To achieve this, we go beyond the standard limitations of formal learning by providing a combination of content, design and technology modeled along the lines of edutainment apps.

The visual representations and interactive narratives in Mathina are developed for modern digital tools, which are most familiar to young learners. The design of the interactive narratives takes both the interactions between children, teachers and parents and the interactions between children and the digital platform into account, which serves as Mathina's learning environment.

Creating a friendly user interface and appealing visual identity for kids: Today's kids spend a lot of time using digital media, and their cognitive and physical limitations present many challenges to them when they do so. We took into account various ways of how today's kids use the web. In the Mathina project we decided to design a user interface that empowers children and make them smarter. Mathina's visual identity takes these complex objectives into account as well as represents gender equity and openness towards cultural differences.

Illustration and animation: In order to create additional visual interest and draw attention towards the content, we implemented exciting illustrations and animations. We believe the animated visuals add a lot of value to the Mathina project and make it much more unique and interactive.

User interaction guidelines for different age groups: The user interface design for Mathina's world is based on several key characteristics of interactive storytelling applications. Such systems, first of all, should encourage interactive learning. Narrative systems have the potential to impact young learners because stories are important in their lives and stories can facilitate learning.

Kids under the age of 7 mostly use touchscreens and apps, while kids above the age of 7 slowly start using a browser to access websites and web applications. Kids above the age of 12 slowly accommodate to more adult-like interaction patterns. Ultimately, our goal is to expose kids to quality content. The guidelines summarized below are tailored to a wide range of learners from the ages of 4 up to 19.

General guidelines for Mathina's learning environment:

1. SIMPLE DATA ENTRY

Interacting with an interface is a physical skill. And because we know that fine motor skills don't fully develop until the age of 10, to an 8-year-old, typing is difficult. For kids under the age of 10, we simplified the keyboard-based data-entry requirements as much as possible.

2. BUTTONS RATHER THAN TEXT LINKS

The youngest children can't control a mouse or a trackpad, but even a 2-year-old can navigate on a tablet. Most 6-year-olds can use a computer mouse, but for the past three years of their lives, they've mostly been playing with tablets. They are used to buttons being the main navigation triggers. Kids under the age of 10 need larger trigger points than older kids.

3. ICONS AND IMAGES

Icons should have a clear connection to the real world, because children have not developed a lot of mental models for abstract icons yet. The younger the user group, the more direct this connection needs to be. Moreover, kids having trouble with reading can navigate more easily by looking at the images.

4. DESIGN FOR SOCIAL INTERACTION

Social behavior doesn't necessarily occur when people are physically in the same room. Children use computers in a social way; they play together and watch each other play. Social setting can also mean uploading videos to YouTube or playing a game on a server.

3 Green, M.C., Strange, J.J. and Brock, T.C. (Eds.) (2002) Narrative Impact: Social and Cognitive Foundations, Lawrence Erlbaum Associates, Mahwah, New Jersey.

5. PROVIDING INFORMATION AND INSTRUCTIONS "JUST IN TIME" AND "ON DEMAND"

Young kids (up to 8) have a "learning by exploration" behavior, but they might leave if they are not immediately successful. They will benefit from "on-demand" instructions. Older kids, on the other hand, benefit from post-failure messages, and they won't be discouraged in the same way if they make mistakes.

6. GAMIFICATION

Kids like playing games. They turn pretty much everything into games and competitions. Part storybook, part game, Mathina's world is loaded with creative gameplay. With every story, a new scenario invites young learners to explore and play games associated with specific math-themed chapters. The scenes change with each story, from commonplace to exotic lands. The main objective is to intertwine games with the storybook, so the young learners will be engaged in constant play.

The project team is dedicated to the idea that quality educational resources are the tools of the future, but they need to be used responsibly as it is essential to focus on thoughtfully spent screen time for our young learners - now more than ever. With that said, all content and inputs provided by the project partners will be free to use under a Creative Commons license.

1.4 Mathina Project's International Consortium

IMAGINARY gGmbH

IMAGINARY is a non-profit international organisation for the communication of modern mathematics, based in Berlin. It started in 2008 at the Mathematics Research Institute from Oberwolfach, and since then its exhibitions have been displayed more than 300 times in 60 countries, reaching more than 2 million visitors, thanks to all content being offered under open licenses at the IMAGINARY platform (www.imaginary.org). IMAGINARY creates math exhibitions, supports education projects, gives infrastructure to science museums, organizes conferences, and offers services to the community of math popularization, with partners such as the UNESCO, several nationwide and international mathematical societies, or the German Federal Ministry of Education and Research. IMAGINARY is the lead partner of Mathina, and brings in its expertise in project management, software development, and designing interactive modules for the public and for non-formal education settings.
https://about.imaginary.org/

Associação Atractor

Atractor is a Portuguese non-profit association. The main goal of Atractor is to attract people to mathematics, aiming to reach a public as wide as possible. To this end, it has been using both physical and virtual interactive contents. Some of Atractor's main activities include: 1) the organization of a large exhibition entitled Matemática Viva (Maths Alive), which was on display at the Pavilion of Knowledge, Lisbon, with huge success; 2) the publication of the DVD Symmetry – the dynamical way, produced in 16000 copies; 3) the publication of the interactive programs AtrMini (www.atractor.pt/mat/AtrMini) and GeC-

la (www.atractor.pt/mat/GeCla); 4) developing and providing a website, with more than 2000 web pages, for math outreach, highly illustrated with images and interactive applications.
https://www.atractor.pt/index-_en.html

Bragi Vizualne Komunikacije
Bragi is a creative agency based in Celje, Slovenia, operating both locally and internationally in the areas of branding, design and development. Some of Bragi's most notable development projects and achievements are: Skillshot the Hunt, a mobile game fully developed in-house, Comenius EduMedia Seals of Approval received in 2009 and 2011 for e-learning programs, as well as Seal of Excellence awarded in 2016 for project Fieldify.
https://bragi.si/en/

Curvilinea Società Cooperativa
Curvilinea is a cooperative company based in Italy. Curvilinea's goal is to popularize mathematics in a fun and playful way, to improve problem-solving abilities of learners. As its main activity, Curvilinea brings laboratorial and interactive activities to classrooms, and contributes to cultural institutions' programs and other cultural events. Curvilinea has also developed an exhibition about cryptography.
https://www.curvilinea.org/?lang=en

Experience Workshop ay
Experience Workshop is a STEAM (Science, Technology, Engineering, Arts and Mathematics) education expert company based in Finland. Its main areas of operation are research, consultancy, project management and educational toolkits in STEAM education and multidisciplinary learning. Experience Workshop develops local and international projects for schools, science centers, universities, cultural and scientific institutions. The goal is to offer opportunities for everyone to learn mathematics through the arts, and to create art through mathematics.
www.experienceworkshop.org

2 MATHINA'S STORIES AND GAMES

2.1 Development of logical thinking in mathina's world: take a trip to logi-city!

The most populated place in Mathina's world is certainly Logi-city. Travelers here find puzzles embedded in stories, and stories embedded in puzzles. Solving the riddles and mysteries, participating in the adventures require clever, logical thinking. Regardless of the variety of different stories, puzzles and activities offered by Logi-city, all can be deciphered by anyone ready to open their minds for something new.

Entering into the story world of Logi-city and dealing with the plethora of puzzles will improve logical thinking and problem-solving skills. If we devote time, attention and encouragement, and let Logi-city's adventurers find the right move by themselves, it will strengthen their independence and perseverance.

Efficient mathematics education is based on learning how to look at the world with an open mind, and how to pose questions when interpreting information or a text. Our verbal skills, such as text comprehension, are strongly linked to logic and need to be constantly refined. The solution to a problem depends on describing and understanding it properly. If I understand, if I manage to clarify the obscure parts, I will come to a solution. Mathematical problem-solving models this, as we decipher what the solution means. Recognizing information, understanding words, sentences, going under the surface... this is what our stories can help in.

The storytellers for the youngest ages shall give a little help only, never more than what is minimally needed. Don't tell the solution, let kids find the next step on their own, but be ready to offer a little more. Lead them with questions instead of instructions. Let kids find possible answers, and discuss with them why all answers are exciting, and why a certain answer is good, and why another one is even better. If children's solutions come from their own experience, it will be more permanent than hearing something from an adult.

Logi-city's stories are not only puzzles, but they are also games. Yet none of these stories, puzzles, and games are carved in stone. Let's change them, make them better if you want. Make them easier or more complex, let children play with the thought and let thoughts play with the children. The stories, puzzles and games included can be thought further if kids require more.

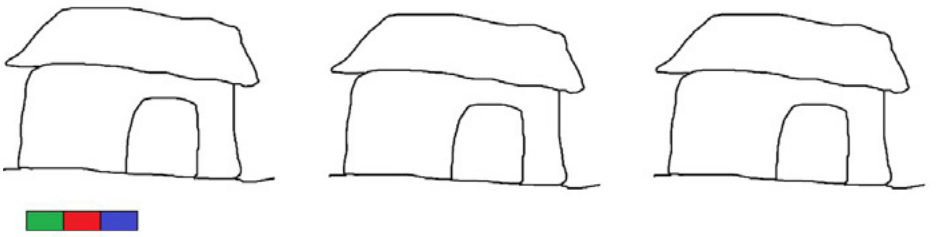
By immersing into these stories, working on puzzles and participating in games, children may find that playing with ideas is a good thing and that they do not necessarily need any expensive equipment to start with. Kids are all different, and we see that almost anything can be turned into a game, es-

pecially into a game of new ideas. The pace of development and growth of thinking follows an infinite number of paths, and varies from child to child.

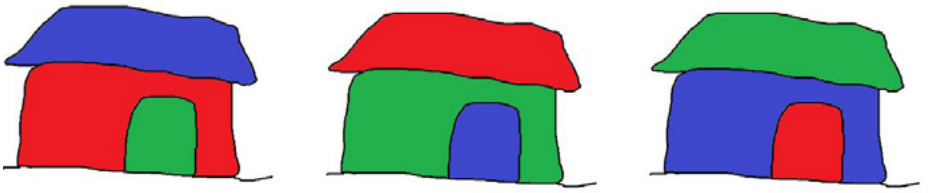
Games and riddles develop multiple abilities of children in a complex way. Memory, because you have to keep in mind the rule. Understanding, because the rules and the essence of the game have to be understood. Adaptation to others. Social sensitivity, because you have to keep in mind the pace and sensitivity of others. Games teach self-discipline, equity, and resilience, even when it comes to the management of the success you achieve.

To give an example for the **age group of 4-6-years:**

Example:
Here are 3 houses next to each other in Logi-city's suburbs, and they wait to be painted by the new tenants.
Green, red and blue paint is there ready for use. The houses have a door, walls and a roof. Each of the colors should be used on all houses but all parts of the house should be of a different color.

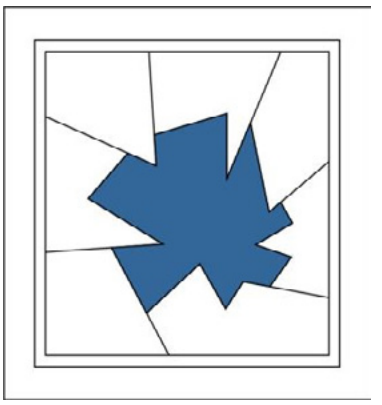


One possible solution:



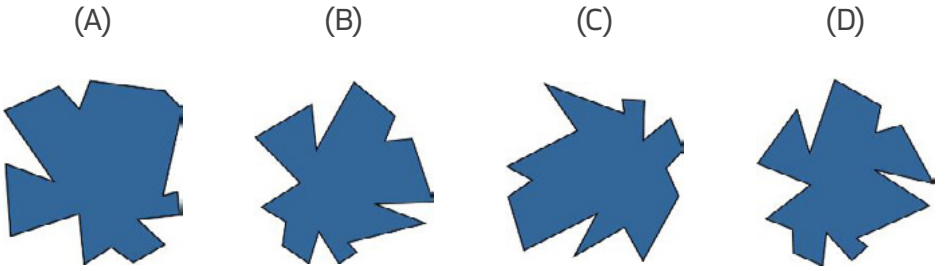
Children around the **age of 7-10** are already able to compare sets. They find the way through a labyrinth, they are able to identify yes and no statements, recognize similarities, differences and shared characteristics.

Example:
The window broke in Logi-city's sports hall, and you are looking for the missing piece to repair the window.



4 Source of the exercise: Звягинцева Л.П., Нацкевич Ю.А., Никифорова Л.В., Титова М.А., Чесакова М.Г.: Международный Конкурс-игра по математике «Слон». Омск, 2012 (downloadable from: https://shihova.ucoz.ru/slon/list_1_zadanija_2_klass.pdf).

Which one is the missing piece?



Around the **age of 11-14**, children are able to explore cause-effect relationships, they can draw multi-step conclusions and are experienced in solving tasks which involve truths and lies. Thinking aids like the pigeon-hole principle or backwards reasoning are already built into their way of thinking.

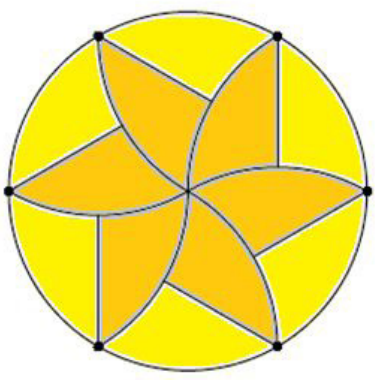
Example:
Andy Logic, the manager of Logi-city's candy company decides to give away all the candies from some of their storage houses. He doesn't however want to give away all candies, because in that case all their 100 storage houses would become empty and the company would go bankrupt.

He invents this smart solution:
The Candy Guard walks by the storage houses a 100 times. In the course of his first walk, he turns each and every lock, during his second walk, only that of every second storage houses, during the third one every third, etc. By turning a lock, locked doors open and open doors lock.
Question: The locks of which storage houses will be open after the procedure?

Solution: square numbers are the right answers: candies in the storage houses nr. 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100 are given away.

We can typically expect from a 15-19+ year old to be able to draw complex logical conclusions, to argue for their right, and get acquainted with indirect reasoning. They see the difference between precondition and consequence, they can handle problems by analogue and inductive thinking. They are already experienced in combinatorics, can model problem situations, and use graphs if needed.

Example:
There is this beautiful pattern on the facade of Logi-city's city hall building:

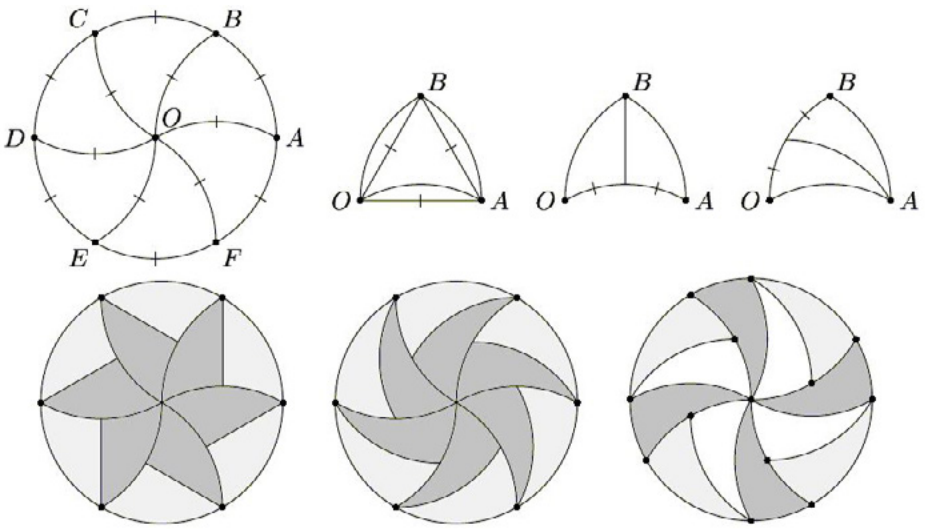


5 Source of the exercise: ВВТихомирова (ed.): Московские математические олимпиады 1993—2005 г. Издательство МЦНМО, 2006

How many components is this pattern made of? What can you say about these components? (Answer: the yellow and the orange regions are of the same size and shape)

Some of Logi-city's residents would like to see different patterns on their own houses, which are however composed according to the same rules like the above one. So they order those from Logi-city's famous architecture office. The architects need your help in solving the following problem: how is it possible to divide this circle-shaped plate into equal shaped and sized parts so that some of the parts neither contain, nor pass through the center point of the circle? Give a different solution than the above one. You are free to create a pattern divided into smaller regions than in the example and including more colours, as long as the basic rule (same size regions, some of them not crossing the central point) is true.

Solution:
Two solutions are included here besides the original example:

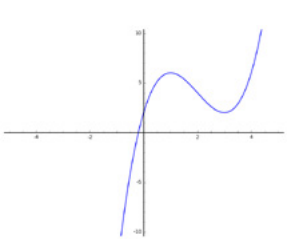


2.2 Developing Spatial Visualization: How to train your Bird of Fire

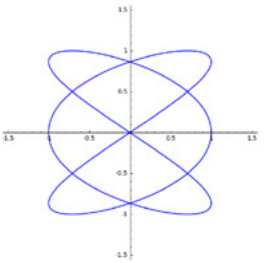
Mathematics is often presented as a language, a way for expression and action of the characters in the story. Someone wants to count a certain number of sheep and chickens, or to measure a rectangular corn field. At some point in your math education, numbers, equations, symbols, etc, become the mathematical actors of the story, and mathematics becomes not only a language but a story in itself. Getting familiar with that mathematics story builds an intuition that helps understanding the ideas behind, and learning more complex ideas to come.

The interplay between an algebraic formula (or function) and its graphical representation is a fundamental topic in mathematics, and a perfect example of such a story relating different mathematical characters (the formulas and the graphs). By giving values to a formula, we can obtain a graphic, and conversely, a graphic can be seen as the set of coordinate values that satisfy a certain formula. Both are two faces of the same entity. Exploring the relationship

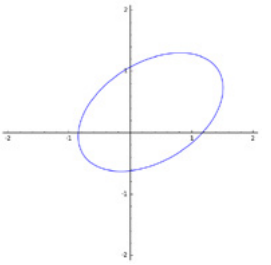
between an algebraic expression and the corresponding graphical image is key to building an intuition of what mathematics is about. It is crucial to exercise the imagination of a world where graphics and shapes live, and those shapes can be described better with mathematical language (algebra) than with common words.



$y = x^3 - 6x^2 + 9x + 2$



$\begin{cases} x = \sin(3t) \\ y = \sin(2t) \end{cases}$



$6x2 - 6xy + 9y2 - 2x - 4y - 6 = 0$

Three ways of describing a curve in the plane:
by an explicit function, by parametric equations, or by an implicit function.

We exploit that perspective in "The land of birds of fire". In this land, there are different types of fire birds, fiery flying creatures (phoenixes, dragons, firebugs) made of flesh and fire, that perform different types of flights in the air, leaving different traces of colorful sparks and smoke suspended in the air, drawing some shapes. These magical animals are fairly intelligent; they can be trained and told how to fly, but they speak instead of a common human language their own language, which is based on mathematical formulas. The mathematical notation (algebraic expressions) is the bridge that allows a human to talk with the flying creatures, and thus to draw and see interesting shapes in the sky.

Throughout the story, the adventurer proceeds from the ideas of direction and movement to those of vector function, equation, derivative... The phoenixes can only fly forward, thus corresponding to functions y=f(x) in mathematics. Dragons can fly also backwards and thus they correspond to parametric curves x=f(t), y=g(t). Firebugs are a swarm of points that position themselves wherever a condition is satisfied, thus corresponding to implicit equations F(x,y)=0. The stories present naturally different animals that fly with different languages, but all describe curves in the air. At this stage, the space is only two-dimensional, watching the birds fly in the sky over the horizon. In a more advanced story, curves and surfaces in three dimensions are also explored.

A surface in the space (Cayley cubic), given by an implicit equation.

Mathematical concepts arise naturally from the features of the story. The direction of flight brings in vectors. Relations between the distance flown and the height bring functions. Rising and descending birds bring growth rate, which introduces derivatives. Bugs located at any place that satisfies a condition bring implicit equations. By adding an interactive app for the visualization of this story, all these ideas can be understood without mathematical formalism. We aim to bring an early exposure of children to mathematical ideas, in an intuitive and visual way, which can be complemented by more formal education.

For the age group 4-6, the core ideas are directions and vectors. In a game, the child controls graphically the direction vector and the curve is drawn. Straight lines are given an important role. This game-oriented control style can reinforce the causational relationship between vectors and curves.

Targeting the age group 7-10, the idea is to start using an intuitive numerical input, instead of just a graphical one. The core ideas are the coordinate system and the relationship between geometric properties and numeric properties. Still, the apps are goal-oriented games embedded in a story.

Around the age 11-14, children are able to use symbolic expressions (functions and equations) to describe curves, and get exposed to the ideas of growth rate and derivative. Stories and apps are more open-ended, giving more options to explore.

The programs **for the age group 15-19+** will be seen as tools to draw any (polynomial) curve by the three methods (explicit, parametric, implicit), and some 3D graphics. Challenges will guide the way to more advanced mathematical ideas.

The above description in terms of age-groups is just a recommendation or an entry-level suggestion. The platform's visitors will be able to explore and play on all levels. Thus, children are encouraged to enter the more advanced chapters as well, to read the stories and play with the applets. Even if the concepts are more advanced than expected, experimentation is always fruitful. On the other hand, chapters designed for younger users can be read and played with as well.

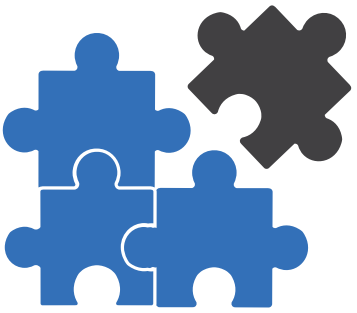
2.3 Break the Secret Code of Mathematics: Mathina's Adventures on Buccaneer Island

Communication is at the base of our society. Aristotle observed that "Man is by nature a social animal", and we can see how much communication is linked to this idea by looking at the way he presented many of his writings: in dialogues. Communicating is a shared experience and involves so many aspects, that the final act of transferring knowledge looks like a small tip of a huge iceberg. This is one of the reasons why cryptography has been included in Mathina's world: it is the branch of mathematics which studies the techniques to encript and decript information. Our world depends on the need of trans-

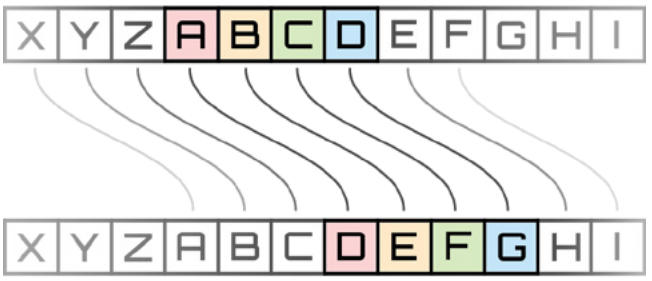
ferring information and on communication.

Playing with cryptography is a way to discover mathematics from a special perspective. Experimenting with secret codes and hidden algorithms often leads to questions like "What am I really communicating by using this message?" or "What is the common knowledge to all the people playing this game?" This kind of experience fuels the motivation to learn more about mathematics.

Many cryptographic methods rely on what mathematicians call "one-way functions". They bear similarities to the combination lock: if it's open, it's very easy to close it. If it's closed, it's always possible to open it, but it's quite time-consuming, if you don't know the correct combination. Cryptography relies on this kind of operation: it's very easy to do it, but very complex to undo it.



An example of a one-way function: solving a puzzle requires more effort than breaking it into pieces.



In the Caesar cypher each letter is shifted by 3 places to the right, according to the alphabet. In this case the operation allowing to decode the message, is as easy as encoding: it's just a shift by 3 places to the left.

Buccaneer Island in Mathina's world is filled with tales about cryptography. Decoding secret messages, finding hidden treasures allow to learn concepts of cryptography while playing and being immersed in an imaginary world. All stories and puzzles rely on interaction and dialogue: sometimes with inhabitants of the island, sometimes by finding or sending messages in bottles and by deciphering hidden writing.

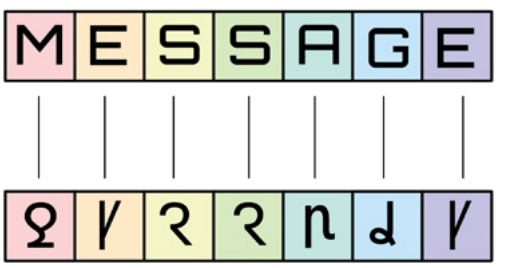
This island is full of secrets. It is believed that Buccaneer Island was chosen as a hideout by one of the best crews of pirates in the world in ancient times. It is no surprise that strange signs, in code languages of that time, are still there to be discovered and interpreted. Rumor has it that pirates' descendants still live

on the island among other people. These playful folks are easy to recognize: they communicate in code language and are always ready to invent strange but fun games.

The stories of Buccaneer Island are based on simple cyphers (often classical ones going back to the time when cryptography was "invented"), which contain all the elements required to understand cryptography. Presenting classical cyphers also gives the opportunity to talk about the evolution of cryptography, hence to find links with history and subjects other than mathematics. The tales of Buccaneer Island follow the modern point of view of cryptography: the procedure used to cypher a message must be considered known to everyone, with the exception of the piece of data called key, which is kept secret and which allows deciphering the message.

For the age group 4-6, the focus will be on the communication of the message and the concept of exchanging information, rather than on a specific cypher. The idea is to introduce the concept of abstraction, in particular in terms of choosing a symbol instead of a specific message. For instance, if we want to exchange information about where to meet in the playground, we can choose a symbol to represent the possible places, and then use it to communicate. In this example, the concept of "the place where to meet" is abstractly replaced by the chosen symbol. This simple yet very important operation comes naturally in the act of playing and speaking. The stories will make it explicit, helping the reader to experience the role of mathematics in this process.

The stories meant for the **age group 7-10** describe a simple cypher, so that kids can begin to experiment with the idea of method and key. They will also experience the time difference between decoding a message with and without the key.



The apparently simple idea that a character, or an entire word, or even an entire sentence can be abstracted as a symbol is one of the basic ideas of cryptography. In the picture, each letter is replaced with a symbol: the basic idea of a substitution cypher.

The activities targeting the **age group 11-14** are more difficult, and introduce more questions. Does repeating a cypher increase its security? Is there a way to decipher a message faster than trying all the possible keys?

The concept of one-way function will be explored more explicitly with the **age group 15-19+**. The structure of the stories and apps allows users to interpret cyphers already introduced in terms of one-way functions. This concept is of prime importance in modern cryptography, for instance it's the basis of public-key methods.

The experiences gathered while wondering in Mathina's world develop curiosity and invite the readers to become part of the story. The riddles contained in the stories are not just problems to solve, but they open new ways to play with different aspects of communication. Let's make a step toward finding the hidden treasure! Let's go ahead, play, experiment and discover by interacting with the tale. Discover the basis of Cryptography and develop your own cypher and share it with your friend to start a new game in real life.

2.4 Exploring symmetry: Mathina at the Symmetry Fair

Most of the symmetry-related activities and challenges in Mathina's world take place at the Symmetry Fair. Explore and enjoy a large number of story-driven games, challenges and activities, in order to train and develop symmetry skills in an interactive and playful way.

The main goal is to help the visitors in Mathina's world to systematize and deepen various concepts arising from their own empirical experience. In a playful and interactive way, you can discover what "symmetry" is and distinguish between different types of symmetry. The strong visual feature of this mathematical subject and its direct relation to the real world makes it an interesting and attractive topic.

The meaning of the word "symmetry" strongly depends on the context in which it arises. In everyday language, this word is generally associated with the existence of a straight line that works as a mirror.

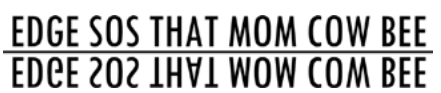
For instance, the words ,

EDGE SOS THAT MOM COW BEE

reflected in a vertical line will result in something like this:

In some cases (like ƎƎDƎ) the result does not even have a letter, and when the result has only letters, it can be a word (MOM) or not (TAHT). Only in the case MOM did we get exactly the same word: that word has a vertical axis of reflection, passing through the middle of the letter O and the figure that represents that word is symmetrical: it does not change when this reflection is applied (it is invariant under this reflection).

Something similar happens to the last word, BEE, when applying a reflection in a horizontal line:



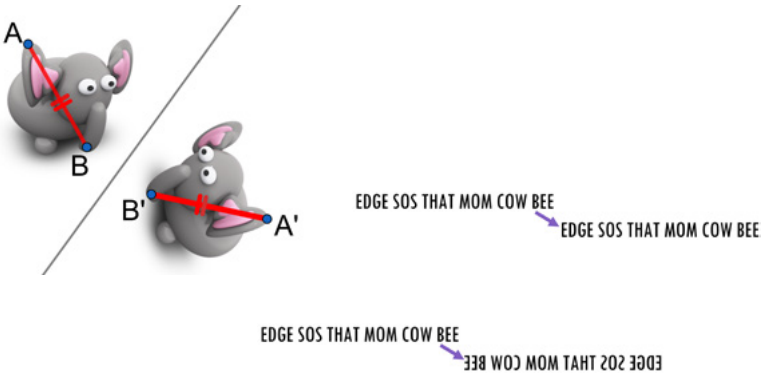
This last word BEE has a horizontal axis of reflection, which passes through the middle of the letters E and B, so that the figure does not change when this reflection is applied to it. In this case it turns out that the reflection of one of the words, MOM, is WOW, a word although different. Among all the words used

up to now, there are only two that are symmetrical: **MOM** and **BEE**. Concerning the word **MOM**, it is important to avoid a confusion: for a word, the conditions "symmetrical with a vertical axis" and "reads the same backward as forward" are not equivalent, as one can see with the word **MADAM**, which is not symmetrical but reads the same in both directions.

What we have pointed out is that in everyday language a (flat) figure is symmetrical when it does not change (it is invariant) under a reflection in a line, which is a transformation of a certain kind. A more general notion of symmetry extends this basic notion, allowing us to contemplate, in addition to straight line reflections, transformations of another kind, for example rotations around a point.

In this example, the only word which is transformed into an identical word is **SOS**, and this word becomes symmetrical, because there is a half-turn around the midpoint of **O** which transforms the word onto itself. And **S**, a letter not symmetrical in the sense of everyday language, is symmetrical within this broader terminology: a 180-degree rotation around the midpoint of **S** leaves **S** the same.

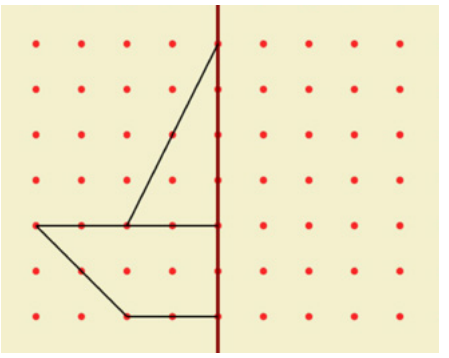
We started the definition of symmetry based on reflections, and extended it to rotations. It is natural to ask whether there are still further extensions and when does such an extension procedure stop. Both types of transformations (reflections and rotations) preserve distances: if two points A and B are transformed into A' and B', then dist(A,B)=dist(A',B'). All extensions still to be considered will correspond to transformations with this same property. How many exist besides the given ones? Just translations and glide-reflections:

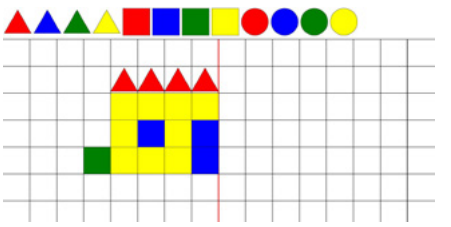


All that we have been talking about is what will be behind the stories planned for the different age levels. We will now give an overview and examples of such stories.
For the **4-6 age-group**, our focus will be on "reflection symmetry". Using a storytelling approach, children will be invited to:

1. choose characters with/without reflection symmetry (the user will be invited to pick, from a given list, the "good" toys, i.e. the toys with reflection axes);

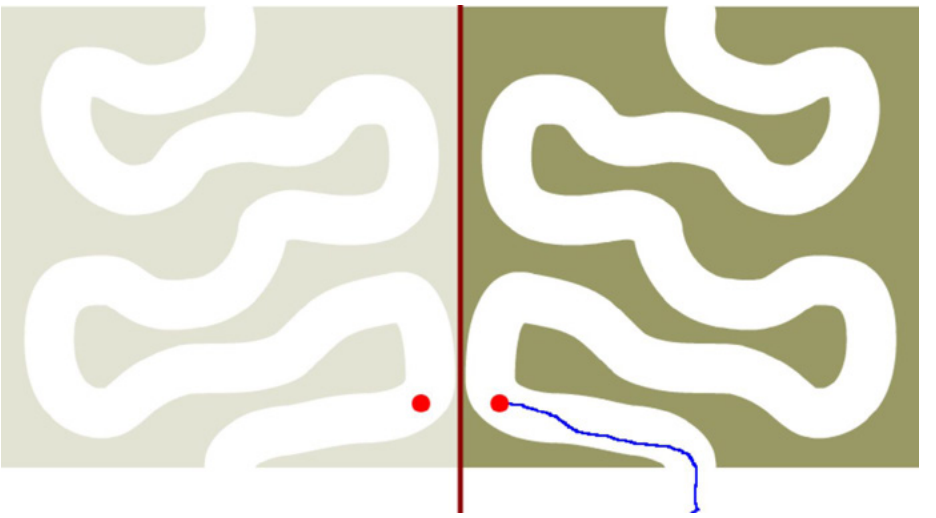
2. On a virtual geoboard players can add segments or polygons, in order to get symmetrical pictures relative to a line.





With regard to the second age-group (7-10), the idea is to further explore the notions of reflection, and extend these explorations to rotation and translation symmetry. Children will be invited to:

solve specially built mazes, based on the use of reflection and rotation symmetry, for example



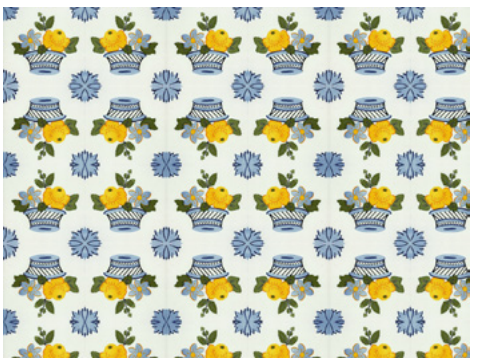
2. draw/classify rosettes and types of friezes. There will be several tools available, e.g. virtual "mirrors" (to simulate reflections) and virtual "rotators",

A rosette [1]

A frieze [1]

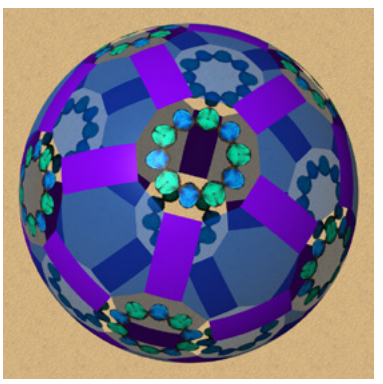
3. find the best ways to draw objects with symmetries.

In what concerns the **age-group 11-14**, the idea is to provide activities similar to the ones already described, as well as more extended ones, concerning the mathematical classification of all the friezes and some wallpaper patterns (including the compositions of reflections and translations, also known as glide reflection).

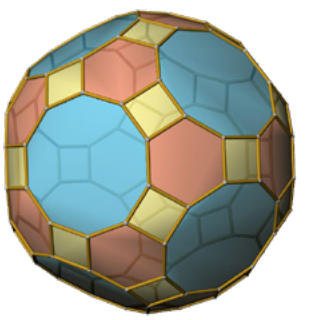


A wallpaper pattern [1]

For the **15-19+ age-group**, the idea is to extend the part related to the classification of friezes to all (17) wallpaper patterns on the plane. Activities related to the exploration of properties concerning spherical symmetry, including symmetries of polyhedra, will also be developed.



Symmetry in the sphere [3,4]



A polyhedron [2]

In line with what was set out in the Mathina project goals, a set of apps will be provided in JavaScript language, which will allow the access from a wide range of devices such as smartphones, tablets and computers.

1 https://www.atractor.pt/mat/GeCla/index-_en.html
2 https://www.atractor.pt/mat/poliedros/pol_index_mdmo-_en.html
3 https://www.atractor.pt/soft-_en.html
4 http://www.geometrygames.org/